# Precalculus

## Concepts Through Functions

## A Right Triangle Approach to Trigonometry

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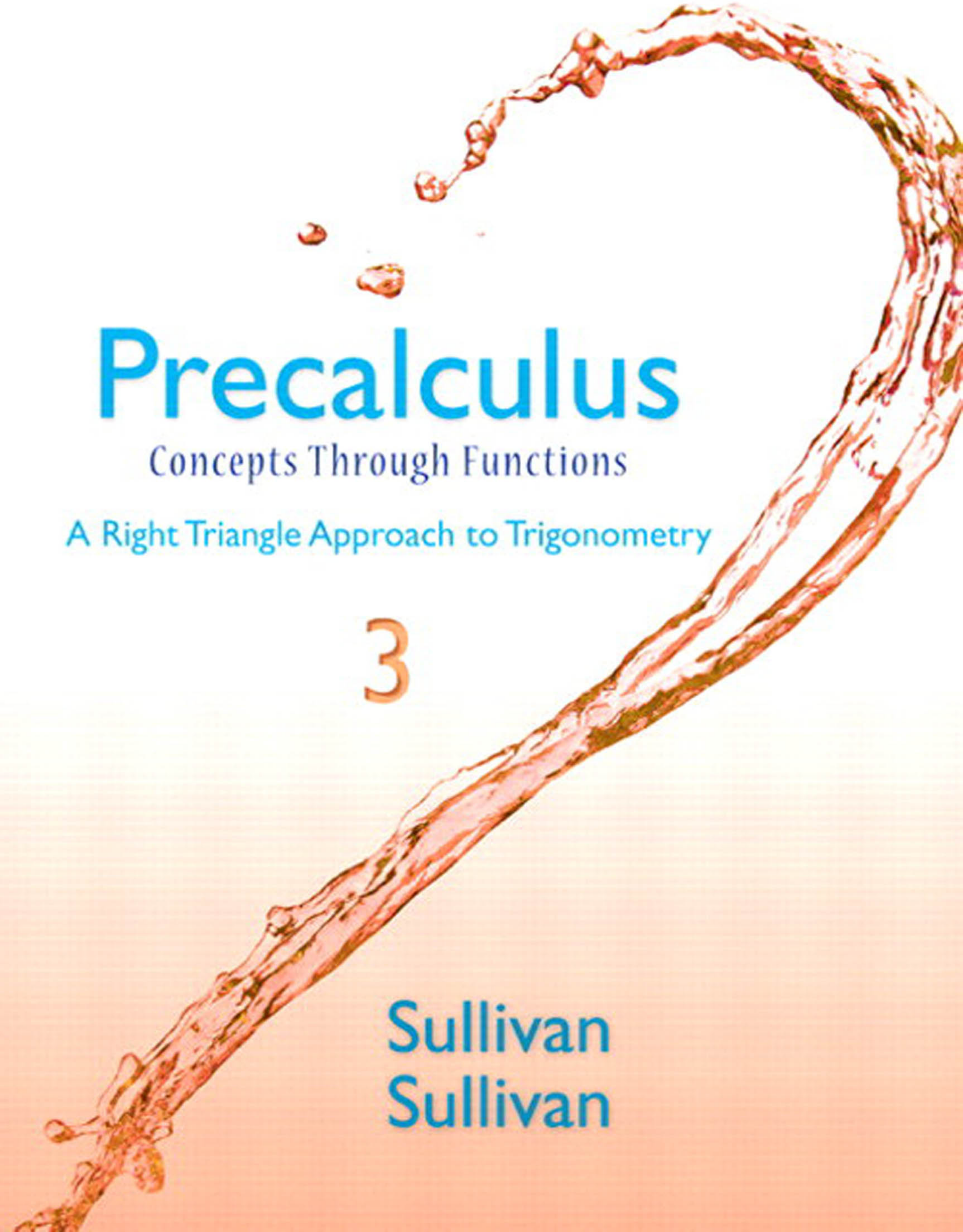

Sullivan
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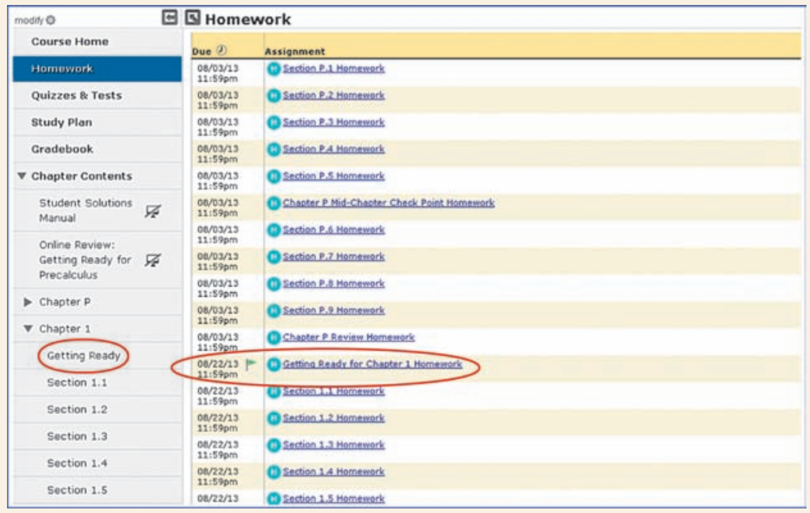
## Getting Ready




Are you frustrated when you know you learned a math concept in the past, but you can't quite remember the skill when it's time to use it? Don't worry!

The authors have included Getting Ready material so you can brush up on forgotten material efficiently by taking a quick skill review quiz to pinpoint the areas where you need help.

Then, a personalized homework assignment provides additional practice on those forgotten concepts, right when you need it.

## Retain Your Knowledge

As you work through your math course, these MyMathLab® exercises support ongoing review to help you maintain essential skills.

The ability to recall important math concepts as you continually acquire new mathematical skills will help you be successful in this math course and in your future math courses.

Scan the code or visit **www.mymathlab.com/success-stories** to learn how students across the country have used MyMathLab to achieve their potential and be successful in their math courses.

# Achieve Your Potential

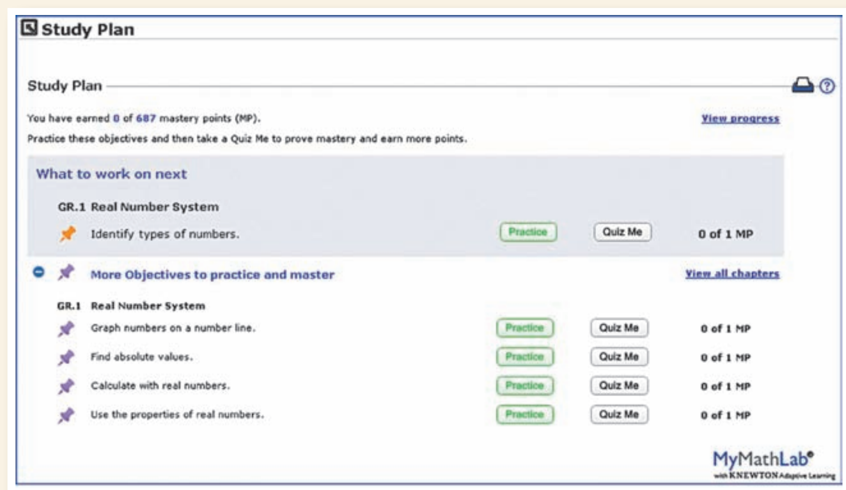
The authors, Michael Sullivan and Michael Sullivan, III, have developed specific content in MyMathLab® to ensure you have many resources to help you achieve success in mathematics - and beyond!
The MyMathLab features described here will help you:

- **Brush up** on math skills and concepts you may have forgotten
- **Retain** new concepts as you move through your math course
- **Develop** skills that will help with your transition to college to your profession

## Adaptive Study Plan

The Study Plan will help you study more efficiently and effectively.

Your performance and activity are assessed continually in real time, providing a personalized experience based on your individual needs.




## Skills for Success

The Skills for Success Module supports your continued success in college.

This module provides tutorials and guidance on a variety of topics, including transitioning to college, online learning, time management and more.

Additional content is provided to help with the development of professional skills such as resume writing or creation and interview preparation.

# Precalculus

## Concepts Through Functions

### A Right Triangle Approach To Trigonometry

## Third Edition

Michael Sullivan
Chicago State University

Michael Sullivan, III
Joliet Junior College

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*For Michael S., Kevin, and Marissa (Sullivan)*

*Shannon, Patrick, and Ryan (Murphy)*

*Maeve, Sean, and Nolan (Sullivan)*

*Kaleigh, Billy, and Timmy (O'Hara)*

*The Next Generation*

# Contents

# To the Student

As you begin, you may feel anxious about the number of theorems, definitions, procedures, and equations. You may wonder if you can learn it all in time. Don't worry, your concerns are normal. This textbook was written with you in mind. If you attend class, work hard, and read and study this book, you will build the knowledge and skills you need to be successful. Here's how you can use the book to your benefit.

## Read Carefully

When you get busy, it's easy to skip reading and go right to the problems. Don't . . . the book has a large number of examples and clear explanations to help you break down the mathematics into easy-to-understand steps. Reading will provide you with a clearer understanding, beyond simple memorization. Read before class (not after) so you can ask questions about anything you didn't understand. You'll be amazed at how much more you'll get out of class if you do this.

## Use the Features

We use many different methods in the classroom to communicate. Those methods, when incorporated into the book, are called "features." The features serve many purposes, from providing timely review of material you learned before (just when you need it), to providing organized review sessions to help you prepare for quizzes and tests. Take advantage of the features and you will master the material.

To make this easier, we've provided a brief guide to getting the most from this book. Refer to the "Prepare for Class," "Practice," and "Review" on pages xxi–xxiii. Spend fifteen minutes reviewing the guide and familiarizing yourself with the features by flipping to the page numbers provided. Then, as you read, use them. This is the best way to make the most of your textbook.

Please do not hesitate to contact us, through Pearson Education, with any questions, suggestions, or comments that would improve this text. We look forward to hearing from you, and good luck with all of your studies.

*Best Wishes!*

*Michael Sullivan*
*Michael Sullivan, III*

# Three Distinct Series

Students have different goals, learning styles, and levels of preparation. Instructors have different teaching philosophies, styles, and techniques. Rather than write one series to fit all, the Sullivans have written three distinct series. All share the same goal—to develop a high level of mathematical understanding and an appreciation for the way mathematics can describe the world around us. The manner of reaching that goal, however, differs from series to series.

## Concepts through Functions Series, Third Edition

This series differs from the others, utilizing a functions approach that serves as the organizing principle tying concepts together. Functions are introduced early in various formats. This approach supports the Rule of Four, which states that functions are represented symbolically, numerically, graphically, and verbally. Each chapter introduces a new type of function and then develops all concepts pertaining to that particular function. The solutions of equations and inequalities, instead of being developed as stand-alone topics, are developed in the context of the underlying functions. Graphing utility coverage is optional and can be included or excluded at the discretion of the instructor: *College Algebra; Precalculus, with a Unit Circle Approach to Trigonometry; Precalculus, with a Right Triangle Approach to Trigonometry*.

## Contemporary Series, Ninth Edition

The Contemporary Series is the most traditional in approach yet modern in its treatment of precalculus mathematics. Graphing utility coverage is optional and can be included or excluded at the discretion of the instructor: *College Algebra, Algebra & Trigonometry, Trigonometry, Precalculus.*

## Enhanced with Graphing Utilities Series, Sixth Edition

This series provides a thorough integration of graphing utilities into topics, allowing students to explore mathematical concepts and foreshadow ideas usually studied in later courses. Using technology, the approach to solving certain problems differs from the Concepts or Contemporary Series, while the emphasis on understanding concepts and building strong skills does not: *College Algebra, Algebra & Trigonometry, Precalculus*.

# Preface to the Instructor

As professors at both an urban university and a community college, Michael Sullivan and Michael Sullivan, III, are aware of the varied needs of Precalculus students, ranging from those who have little mathematical background and a fear of mathematics courses, to those having a strong mathematical education and a high level of motivation. For some of your students, this will be their last course in mathematics, whereas others will further their mathematical education. This text is written for both groups.

As a teacher, and as an author of precalculus, engineering calculus, finite mathematics, and business calculus texts, Michael Sullivan understands what students must know if they are to be focused and successful in upper-level math courses. However, as a father of four, he also understands the realities of college life. As an author of a developmental mathematics series, Michael's co-author and son, Michael Sullivan, III, understands the trepidations and skills students bring to the Precalculus course. Michael, III also believes in the value of technology as a tool for learning that enhances understanding without sacrificing math skills. Together, both authors have taken great pains to ensure that the text contains solid, student-friendly examples and problems, as well as a clear and seamless writing style.

A tremendous benefit of authoring a successful series is the broad-based feedback we receive from teachers and students. We are sincerely grateful for their support. Virtually every change in this edition is the result of their thoughtful comments and suggestions. We are sincerely grateful for this support and hope that we have been able to take these ideas and, building upon a successful first edition, make this series an even better tool for learning and teaching. We continue to encourage you to share with us your experiences teaching from this text.

## About This Book

This book utilizes a functions approach to Precalculus. Functions are introduced early (Chapter 1) in various formats: maps, tables, sets of ordered pairs, equations, and graphs. Our approach to functions illustrates the symbolic, numeric, graphic, and verbal representations of functions. This allows students to make connections between the visual representation of a function and its algebraic representation.

It is our belief that students need to "hit the ground running" so that they do not become complacent in their studies. After all, it is highly likely that students have been exposed to solving equations and inequalities prior to entering this class. By spending precious time reviewing these concepts, students are likely to think of the course as a rehash of material learned in other courses and say to themselves, "I know this material, so I don't have to study." This may result in the students developing poor study habits for this course. By introducing functions early in the course, students are less likely to develop bad habits.

Another advantage of the early introduction of functions is that the discussion of equations and inequalities can focus around the concept of a function. For example, rather than asking students to solve an equation such as $2x^2 + 5x + 2 = 0$, we ask students to find the zeros of $f(x) = 2x^2 + 5x + 2$ or solve $f(x) = 0$ when $f(x) = 2x^2 + 5x + 2$. While the technique used to solve this type of problem is the same, the fact that the problem looks different to the student means the student is less apt to say, "Oh, I already have seen this problem before, and I know how to solve it." In addition, in Calculus students are going to be asked to solve equations such as $f'(x) = 0$, so solving $f(x) = 0$ is a logical prerequisite skill to practice in Precalculus. Another advantage to solving equations through the eyes of a function is that the properties of functions can be included in the solution. For example, the linear function $f(x) = 2x - 3$ has one real zero because the function $f$ is increasing on its domain.

## Features in the Third Edition

Rather than provide a list of new features here, that information can be found on pages xxi–xxiii.

This places the new features in their proper context, as building blocks of an overall learning system that has been carefully crafted over the years to help students get the most out of the time they put into studying. Please take the time to review the features listed on pages xxi–xxiii and to discuss them with your students at the beginning of your course. Our experience has been that when students utilize these features, they are more successful in the course.

## New to the Third Edition

- **Retain Your Knowledge** This new category of problems in the exercise set are based on the article "To Retain New Learning, Do the Math" published in the Edurati Review in which author Kevin Washburn suggests that "the more students are required to recall new content or skills, the better their memory will be." It is frustrating when students cannot recall skills learned earlier in the course. To alleviate this recall problem, we have created "Retain Your Knowledge" problems. These are problems considered to be "final exam material" that students must complete to maintain their skills. All the answers to these problems appear in the back of the book and all are programmed in MyMathLab.
- **Guided Lecture Notes** Ideal for online, emporium/redesign courses, inverted classrooms or traditional lecture classrooms. These lecture notes assist students in taking thorough, organized, and understandable notes as they watch the Author in Action videos by asking students to complete definitions, procedures, and examples based

on the content of the videos and book. In addition, experience suggests that students learn by doing and understanding the why/how of the concept or property. Therefore, many sections will have an exploration activity to motivate student learning. These explorations will introduce the topic and/or connect it somehow to either a real world application or previous section. For example, when teaching about the vertical line test in Section 1.2, after the theorem statement, the notes ask the students to explain why the vertical line test works by using the definition of a function. This helps students process the information at a higher level of understanding.

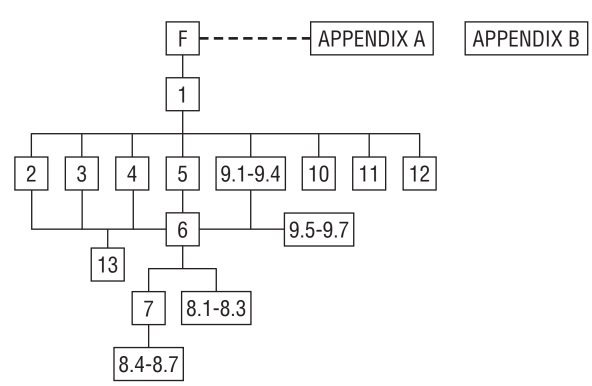
- **Chapter Projects,** which apply the concepts of each chapter to a real-world situation, have been enhanced to give students an up-to-the-minute experience. Many projects are new and Internet-based, requiring the student to research information online in order to solve problems.
- **Author Solves It MathXL Video Clips**—author Michael Sullivan, III solves MathXL exercises typically requested by his students for more explanation or tutoring. These videos are a result of Sullivan's experiences in the classroom and experiences in teaching online.
- **Exercise Sets** at the end of each section remain classified according to purpose. The "*Are You Prepared?*" exercises have been expanded to better serve the student who needs a just-in-time review of concepts utilized in the section. The *Concepts and Vocabulary* exercises have been updated. These fill-in-the-blank and True/False problems have been written to serve as reading quizzes. *Skill Building* exercises develop the student's computational skills and are often grouped by objective. *Mixed Practice* exercises have been added where appropriate. These problems offer a comprehensive assessment of the skills learned in the section by asking problems that relate to more than one objective. Sometimes these require information from previous sections so students must utilize skills learned throughout the course. *Applications and Extension* problems have been updated and many new problems involving sourced information and data have been added to bring relevance and timeliness to the exercises. The *Explaining Concepts: Discussion and Writing* exercises have been updated and reworded to stimulate discussion of concepts in online discussion forums. These can also be used to spark classroom discussion. Finally, in the **Annotated Instructor's Edition,** we have preselected problems that can serve as sample homework assignments. These are indicated by a blue underline, and they are assignable in MyMathLab® as part of a "Ready-to-Go" course, if desired.
- The **Chapter Review** now includes answers to all the problems. We have created a separate review worksheet for each chapter to help students review and practice key skills to prepare for exams. The worksheets can be found within MyMathLab® or downloaded from the Instructor's Resource Center.

## Changes in the Third Edition

- **CONTENT**
  - **Chapter 2, Section 4** A new objective "Find a quadratic function given its vertex and one point" has been added.
  - **Chapter 2, Section 5** A new example was added to illustrate that quadratic inequalities may have the empty set or all real numbers as a solution.
  - **Chapter 3, Sections 1 and 4** The content related to describing the behavior of the graph of a polynomial or rational function near a zero has been removed.
  - **Chapter 3, Section 4** Content has been added that discusses the role of multiplicity and behavior of the graph of rational function as the graph approaches a vertical asymptote.
- **ORGANIZATION**
  - **Chapter 3, Sections 5 and 6** Section 5, *The Real Zeros of a Polynomial Function* and Section 6, *Complex Zeros, Fundamental Theorem of Algebra* have been moved to Sections 2 and 3, respectively. This was done in response to reviewer requests that "everything involving polynomials" be located sequentially. Skipping the new Sections 2 and 3 and proceeding to Section 4 *Properties of Rational Functions* can be done without loss of continuity.

## Using this Book Effectively and Efficiently with Your Syllabus

To meet the varied needs of diverse syllabi, this book contains more content than is likely to be covered in a typical Precalculus course. As the chart illustrates, this book has been organized with flexibility of use in mind. Even within a given chapter, certain sections are optional and can be omitted without loss of continuity. See the detail following the flow chart.




**Foundations A Prelude to Functions**
Quick coverage of this chapter, which is mainly review material, will enable you to get to Chapter 1, *Functions and Their Graphs*, earlier.

**Chapter 1 Functions and Their Graphs**
Perhaps the most important chapter. Sections 1.6 and 1.7 are optional.

**Chapter 2 Linear and Quadratic Functions**
Topic selection depends on your syllabus. Sections 2.2, 2.6, and 2.7 may be omitted without a loss of continuity.

**Chapter 3 Polynomial and Rational Functions**
Topic selection depends on your syllabus. Section 3.6 is optional.

**Chapter 4 Exponential and Logarithmic Functions**
Sections 4.1–4.6 follow in sequence. Sections 4.7–4.9 are optional.

**Chapter 5 Trigonometric Functions**
The sections follow in sequence. Section 5.8 is optional.

**Chapter 6 Analytic Trigonometry**
Sections 6.2 and 6.7 may be omitted in a brief course.

**Chapter 7 Applications of Trigonometric Functions**
Sections 7.4 and 7.5 may be omitted in a brief course.

**Chapter 8 Polar Coordinates; Vectors**
Sections 8.1–8.3 and Sections 8.4–8.7 are independent and may be covered separately.

**Chapter 9 Analytic Geometry**
Sections 9.1–9.4 follow in sequence. Sections 9.5, 9.6, and 9.7, are independent of each other, but each requires Sections 9.1–9.4.

**Chapter 10 Systems of Equations and Inequalities**
Sections 10.2–10.7 may be covered in any order. Section 10.8 requires Section 10.7.

**Chapter 11 Sequences; Induction; the Binomial Theorem**
There are three independent parts: Sections 11.1–11.3, Section 11.4, and Section 11.5.

**Chapter 12 Counting and Probability**
The sections follow in sequence.

**Appendix A Review**
This review material may be covered at the start of a course or used as a just-in-time review. Specific references to this material occur throughout the text to assist in the review process.

**Appendix B Graphing Utilities**
Reference is made to these sections at the appropriate place in the text.

## Acknowledgments

Textbooks are written by authors, but evolve from an idea to final form through the efforts of many people. It was Don Dellen who first suggested this book and series. Don is remembered for his extensive contributions to publishing and mathematics.

Thanks are due to the following people for their assistance and encouragement to the preparation of this edition:

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Finally, we offer our grateful thanks to the dedicated users and reviewers of our books, whose collective insights form the backbone of each textbook revision.

Our list of indebtedness just grows and grows. And, if we've forgotten anyone, please accept our apology. Thank you all.

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Kenneth D. Reeves—San Antonio College
Elsi Reinhardt—Truckee Meadows Community College
Jose Remesar—Miami Dade College, Wolfson Campus
Jane Ringwald—Iowa State University
Stephen Rodi—Austin Community College
William Rogge—Lincoln Northeast High School
Howard L. Rolf—Baylor University
Mike Rosenthal—Florida International University
Phoebe Rouse—Louisiana State University
Edward Rozema—University of Tennessee at Chattanooga
David Ruffatto—Joliet Junior College
Dennis C. Runde—Manatee Community College
Alan Saleski—Loyola University of Chicago
Susan Sandmeyer—Jamestown Community College
Brenda Santistevan—Salt Lake Community College
Catherine Sausville—George Mason University
Linda Schmidt—Greenville Technical College
Ingrid Scott—Montgomery College
A.K. Shamma—University of West Florida
Martin Sherry—Lower Columbia College
Carmen Shershin—Florida International University
Tatrana Shubin—San Jose State University
Anita Sikes—Delgado Community College
Timothy Sipka—Alma College
Charlotte Smedberg—University of Tampa
Lori Smellegar—Manatee Community College
Gayle Smith—Loyola Blakefield
Leslie Soltis—Mercyhurst College
John Spellman—Southwest Texas State University
Karen Spike—University of North Carolina
Rajalakshmi Sriram—Okaloosa-Walton Community College
Katrina Staley—North Carolina Agricultural
Becky Stamper—Western Kentucky University
Judy Staver—Florida Community College-South
Neil Stephens—Hinsdale South High School
Sonya Stephens—Florida A&M University
Patrick Stevens—Joliet Junior College
John Sumner—University of Tampa
Steve Szabo—Eastern Kentucky University
Matthew TenHuisen—University of North Carolina, Wilmington
Christopher Terry—Augusta State University
Diane Tesar—South Suburban College
Tommy Thompson—Brookhaven College
Martha K. Tietze—Shawnee Mission Northwest High School
Richard J. Tondra—Iowa State University
Suzanne Topp—Salt Lake Community College
Marilyn Toscano—University of Wisconsin, Superior
Marvel Townsend—University of Florida
Jim Trudnowski—Carroll College
Robert Tuskey—Joliet Junior College
Mihaela Vajiac—Chapman University-Orange
Richard G. Vinson—University of South Alabama
Jorge Viola-Prioli—Florida Atlantic University
Mary Voxman—University of Idaho
Jennifer Walsh—Daytona Beach Community College
Donna Wandke—Naperville North High School
Timothy L. Warkentin—Cloud County Community College
Hayat Weiss—Middlesex Community College
Kathryn Wetzel—Amarillo College
Darlene Whitkenack—Northern Illinois University
Suzanne Williams—Central Piedmont Community College
Larissa Williamson—University of Florida
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Canton Woods—Auburn University
Tamara S. Worner—Wayne State College
Terri Wright—New Hampshire Community Technical College, Manchester
George Zazi—Chicago State University
Sharyn Zias—Jamestown Community College
Steve Zuro—Joliet Junior College

*Michael Sullivan*
*Chicago State University*

*Michael Sullivan, III*
*Joliet Junior College*

# Prepare for Class "Read the Book"

| Feature | Description | Benefit | Page |
|---|---|---|---|
| **Every Chapter Opener begins with...** | | | |
| **Chapter Opening Article & Project** | Each chapter begins with a current article and ends with a related project. The article describes a real situation. | The Article describes a real situation. The Project lets you apply what you learned to solve a related problem. | 273, 374 |
| **NEW! Internet Based Projects** | The projects allow for the integration of spreadsheet technology that students will need to be a productive member of the workforce. | The projects allow the opportunity for students to collaborate and use mathematics to deal with issues that come up in their lives. | 273, 374 |
| **Every Section begins with...** | | | |
| **Learning Objectives** 2 | Each section begins with a list of objectives. Objectives also appear in the text where the objective is covered. | These focus your studying by emphasizing what's most important and where to find it. | 294 |
| **Sections contain...** | | | |
| **Preparing for this Section** | Most sections begin with a list of key concepts to review with page numbers. | Ever forget what you've learned? This feature highlights previously learned material to be used in this section. Review it, and you'll always be prepared to move forward. | 294 |
| **Now Work** the **'Are You Prepared?' Problems** | Problems that assess whether you have the prerequisite knowledge for the upcoming section. | Not sure you need the Preparing for This Section review? Work the 'Are You Prepared?' problems. If you get one wrong, you'll know exactly what you need to review and where to review it! | 294, 305 |
| **Now Work PROBLEMS** | These follow most examples and direct you to a related exercise. | We learn best by doing. You'll solidify your understanding of examples if you try a similar problem right away, to be sure you understand what you've just read. | 301, 306 |
| **WARNING** | Warnings are provided in the text. | These point out common mistakes and help you to avoid them. | 328 |
| **Exploration** and **Seeing the Concept** | These represent graphing utility activities to foreshadow a concept or solidify a concept just presented. | You will obtain a deeper and more intuitive understanding of theorems and definition. | 200, 315 |
| In Words | These provide alternative descriptions of select definitions and theorems. | Does math ever look foreign to you? This feature translates math into plain English. | 311 |
| **CALCULUS** | These appear next to information essential for the study of calculus. | Pay attention–if you spend extra time now, you'll do better later! | 70, 302 |
| **SHOWCASE EXAMPLES** | These examples provide "how-to" instruction by offering a guided, step-by-step approach to solving a problem. | With each step presented on the left and the mathematics displayed on the right, students can immediately see how each step is employed. | 204 |
| **Model It!** Examples and Problems | These are examples and problems that require you to build a mathematical model from either a verbal description or data. The homework Model It! problems are marked by purple headings. | It is rare for a problem to come in the form, "*Solve the following equation*". Rather, the equation must be developed based on an explanation of the problem. These problems require you to develop models that will allow you to describe the problem mathematically and suggest a solution to the problem. | 319, 347 |

# Practice "Work the Problems"

| Feature | Description | Benefit | Page |
|---|---|---|---|
| 'Are You Prepared?' Problems | These assess your retention of the prerequisite material you'll need. Answers are given at the end of the section exercises. This feature is related to the Preparing for This Section feature. | Do you always remember what you've learned? Working these problems is the best way to find out. If you get one wrong, you'll know exactly what you need to review and where to review it! | 294, 305 |
| Concepts and Vocabulary | These short-answer questions, mainly Fill-in-the-Blank and True/False items, assess your understanding of key definitions and concepts in the current section. | It is difficult to learn math without knowing the language of mathematics. These problems test your understanding of the formulas and vocabulary. | 305 |
| Skill Building | Correlated to section examples, these problems provide straightforward practice. | It's important to dig in and develop your skills. These problems provide you with ample practice to do so. | 305–307 |
| Mixed Practice | These problems offer comprehensive assessment of the skills learned in the section by asking problems that relate to more than one concept or objective. These problems may also require you to utilize skills learned in previous sections. | Learning mathematics is a building process. Many concepts are interrelated. These problems help you see how mathematics builds on itself and also see how the concepts tie together. | 307–308 |
| Applications and Extensions | These problems allow you to apply your skills to real-world problems. They also allow you to extend concepts learned in the section. | You will see that the material learned within the section has many uses in everyday life. | 308–310 |
| Discussion and Writing | "Discussion and Writing" problems are colored red. These support class discussion, verbalization of mathematical ideas, and writing and research projects. | To verbalize an idea, or to describe it clearly in writing, shows real understanding. These problems nurture that understanding. Many are challenging but you'll get out what you put in. | 310 |
| NEW! Retain Your Knowledge | These problems allow you to practice content learned earlier in the course. | The ability to remember how to solve all the different problems learned throughout the course is difficult. These help you remember. | 310 |
| Now Work PROBLEMS | Many examples refer you to a related homework problem. These related problems are marked by a pencil and orange numbers. | If you get stuck while working problems, look for the closest Now Work problem and refer back to the related example to see if it helps. | 304, 307 |
| Chapter Review Problems | Every chapter concludes with a comprehensive list of exercises to practice. Use the list of objectives to determine the objective and examples that correspond to the problems. | Work these problems to verify you understand all the skills and concepts of the chapter. Think of it as a comprehensive review of the chapter. | 369–372 |

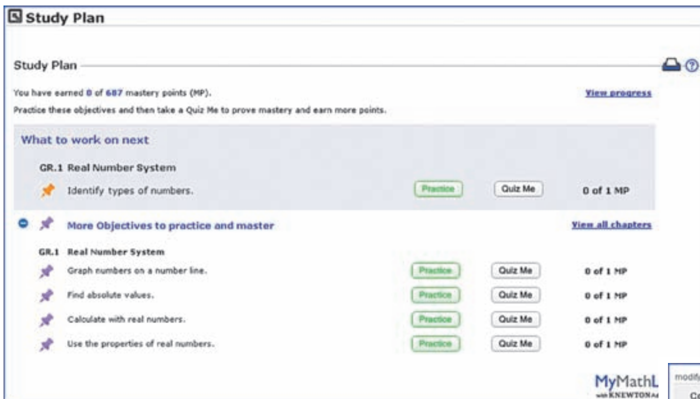
# Review "Study for Quizzes and Tests"

| Feature | Description | Benefit | Page |
|---|---|---|---|
| **Chapter Review at the end of each chapter contains…** | | | |
| **Things to Know** | A detailed list of important theorems, formulas, and definitions from the chapter. | Review these and you'll know the most important material in the chapter! | 367–368 |
| **You Should Be able to…** | Contains a complete list of objectives by section, examples that illustrate the objective, and practice exercises that test your understanding of the objective. | Do the recommended exercises and you'll have mastery over the key material. If you get something wrong, review the suggested page numbers and try again. | 369 |
| **Review Exercises** | These provide comprehensive review and practice of key skills, matched to the Learning Objectives for each section. | Practice makes perfect. These problems combine exercises from all sections, giving you a comprehensive review in one place. | 369–372 |
| Chapter Test | About 15-20 problems that can be taken as a Chapter Test. Be sure to take the Chapter Test under test conditions—no notes! | Be prepared. Take the sample practice test under test conditions. This will get you ready for your instructor's test. If you get a problem wrong, you can watch the Chapter Test Prep Video. | 372–373 |
| Cumulative Review | These problem sets appear at the end of each chapter, beginning with Chapter 2. They combine problems from previous chapters, providing an ongoing cumulative review. | These are really important. They will ensure that you are not forgetting anything as you go. These will go a long way toward keeping you primed for the final exam. | 373 |
| Chapter Project | The Chapter Project applies to what you've learned in the chapter. Additional projects are available on the Instructor's Resource Center (IRC). | The Project gives you an opportunity to apply what you've learned in the chapter to the opening article.If your instructor allows, these make excellent opportunities to work in a group, which is often the best way of learning math. | 374 |
| **NEW! Internet Based Projects** | In selected chapters, a web-based project is given. | The projects allow the opportunity for students to collaborate and use mathematics to deal with issues that come up in their lives. | 374 |

# Resources for Success

## MyMathLab® Online Course (access code required)

MyMathLab delivers **proven results** in helping individual students succeed. It provides **engaging experiences** that personalize, stimulate, and measure learning for each student. And, it comes from an **experienced partner** with educational expertise and an eye on the future. MyMathLab helps prepare students and gets them thinking more conceptually and visually through the following features:

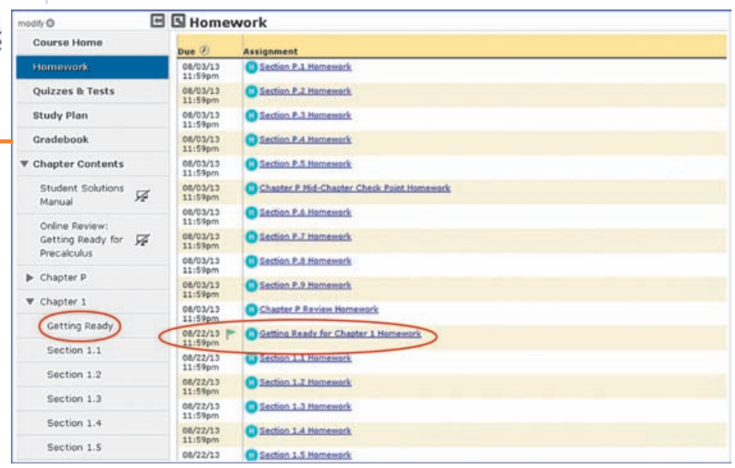


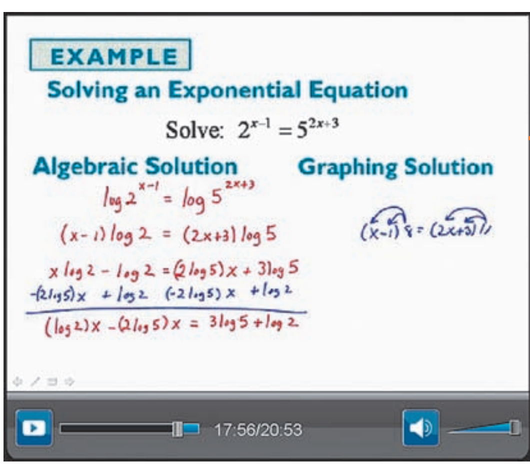

### Adaptive Study Plan

The Study Plan makes studying more efficient and effective for every student. Performance and activity are assessed continually in real time. The data and analytics are used to provide personalized content–reinforcing concepts that target each student's strengths and weaknesses.

### Getting Ready

Students refresh prerequisite topics through assignable skill review quizzes and personalized homework integrated in MyMathLab.



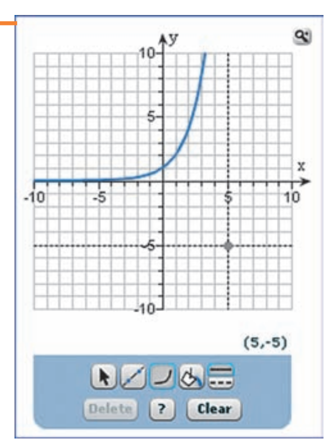




### Video Assessment

Video assessment is tied to key Author in Action videos to check student's conceptual understanding of important math concepts.

### Enhanced Graphing Functionality

**New** functionality within the graphing utility allows graphing of 3-point quadratic functions, 4-point cubic graphs, and transformations in exercises.




**Skills for Success Modules** are integrated within MyMathLab course to help students succeed in collegiate courses and prepare for future professions.

**Retain Your Knowledge** These new exercises support ongoing review at the course level and help students maintain essential skills.

# Instructor Resources

Additional resources can be downloaded from **www.pearsonhighered.com** or hardcopy resources can be ordered from your sales representative.

### Ready to Go MyMathLab® Course

Now it is even easier to get started with MyMathLab. The Ready to Go MyMathLab course option includes author-chosen preassigned homework, integrated review, and more.

### TestGen®

TestGen® (www.pearsoned.com/testgen) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text.

### PowerPoint® Lecture Slides

Fully editable slides that correlate to the textbook.

### Annotated Instructor's Edition

Shorter answers are on the page beside the exercises. Longer answers are in the back of the book.

### Instructor Solutions Manual

Includes fully worked solutions to all textbook exercises.

### Mini Lecture Notes

Includes additional examples and helpful teaching tips, by section.

### Online Chapter Projects

Additional projects that let students apply what was learned in the chapter.

# Student Resources

Additional resources to help student success:

### Lecture Video

Author in Action videos are actual classroom lectures with fully worked out examples presented by Michael Sullivan, III. All video is assignable within MyMathlab.

### Chapter Test Prep Videos

Students can watch instructors work through step-by-step solutions to all chapter test exercises from the textbook. These are available in MyMathlab and on YouTube.

### Student Solutions Manual

Provides detailed worked-out solutions to odd-numbered exercises.

### Guided Lecture Notes

These lecture notes assist students in taking thorough, organized, and understandable notes while watching Author in Action videos. Students actively participate in learning the how/why of important concepts through explorations and activities. The Guided Lecture Notes are available as PDF's and customizable Word files in MyMathLab. They can also be packaged with the textbook and MyMathLab access code.

### Algebra Review

Four Chapters of Intermediate Algebra review. Perfect for a slower-paced course or for individual review.

# Applications Index

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F

# Foundations: A Prelude to Functions

## How to Value a House

Two things to consider in valuing a home are, first, how does it compare to similar homes that have sold recently? Is the asking price fair? And second, what value do you place on the advertised features and amenities? Yes, other people might value them highly, but do you?

Zestimate home valuation, RealEstateABC.com, and Reply.com are among the many algorithmic (generated by a computer model) starting points in figuring out the value of a home. They show you how the home is priced relative to other homes in the area, but you need to add in all the things that only someone who has seen the house knows. You can do that using My Estimator, and then you create your own estimate and see how it stacks up against the asking price.


## Looking at "Comps"

Knowing whether an asking price is fair will be important when you're ready to make an offer on a house. It will be even more important when your mortgage lender hires an appraiser to determine whether the house is worth the loan you're after.

Check with your agent, Zillow.com, propertyshark.com, or other websites to see recent sales of homes in the area that are similar, or comparable, to what you're looking for. Print them out and keep these "comps" in a three-ring binder; you'll be referring to them quite a bit.

Note that "recent sales" usually means within the last six months. A sales price from a year ago may bear little or no relation to what is going on in your area right now. In fact, some lenders will not accept comps older than three months.

Market activity also determines how easy or difficult it is to find accurate comps. In a "hot" or busy market, with sales happening all the time, you're likely to have lots of comps to choose from. In a less active market, finding reasonable comps becomes harder. And if the home you're looking at has special design features, finding a comparable property is harder still. It's also necessary to know what's going on in a given sub-segment. Maybe large, high-end homes are selling like hotcakes, but owners of smaller houses are staying put, or vice versa.

***Source:*** *http://realestate.yahoo.com/Homevalues/How_to_Value_a_House.html*

*—See the Internet-based Chapter Project—*

## <A Look Back

Appendix A reviews skills from Intermediate Algebra.

## A Look Ahead>

Here we connect algebra and geometry using the rectangular coordinate system. In the 1600s, algebra had developed to the point that René Descartes (1596–1650) and Pierre de Fermat (1601–1665) were able to use rectangular coordinates to translate geometry problems into algebra problems, and vice versa. This allowed both geometers and algebraists to gain new insights into their subjects, which had been thought to be separate but now were seen as connected.

## Outline

# F.1 The Distance and Midpoint Formulas

**PREPARING FOR THIS SECTION** *Before getting started, review the following:*

- Algebra Essentials (Appendix A, Section A.1, pp. A1–A10)
- Geometry Essentials (Appendix A, Section A.2, pp. A13–A19)

**Now Work** the **'Are You Prepared?'** problems on page 6.

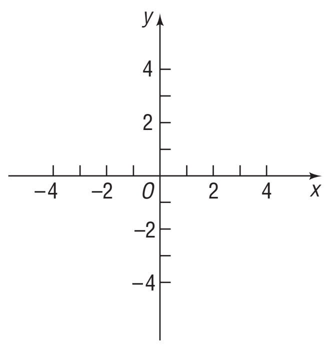
**OBJECTIVES** **1** Use the Distance Formula (p. 3)
**2** Use the Midpoint Formula (p. 5)

## Rectangular Coordinates

A point on the real number line is located by a single real number called the *coordinate of the point*. For work in a two-dimensional plane, points are located by using two numbers.

Begin with two real number lines located in the same plane: one horizontal and the other vertical. The horizontal line is called the ***x*-axis,** the vertical line the ***y*-axis,** and the point of intersection the **origin *O*.** See Figure 1. Assign coordinates to every point on these number lines using a convenient scale. Recall that the scale of a number line is the distance between 0 and 1. In mathematics, we usually use the same scale on each axis, but in applications, a different scale is often used.

**Figure 1**

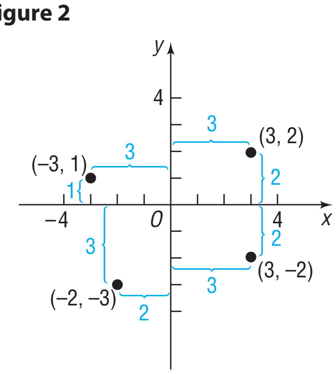



The origin $O$ has a value of 0 on both the $x$-axis and the $y$-axis. Points on the $x$-axis to the right of $O$ are associated with positive real numbers, and those to the left of $O$ are associated with negative real numbers. Points on the $y$-axis above $O$ are associated with positive real numbers, and those below $O$ are associated with negative real numbers. In Figure 1, the $x$-axis and $y$-axis are labeled as $x$ and $y$, respectively, and an arrow at the end of each axis is used to denote the positive direction.

The coordinate system described here is called a **rectangular** or **Cartesian*** **coordinate system.** The plane formed by the $x$-axis and $y$-axis is sometimes called the ***xy*-plane,** and the $x$-axis and $y$-axis are referred to as the **coordinate axes.**

Any point $P$ in the $xy$-plane can be located by using an **ordered pair** $(x, y)$ of real numbers. Let $x$ denote the signed distance of $P$ from the $y$-axis (*signed* means that, if $P$ is to the right of the $y$-axis, then $x > 0$, and if $P$ is to the left of the $y$-axis, then $x < 0$); and let $y$ denote the signed distance of $P$ from the $x$-axis. The ordered pair $(x, y)$, also called the **coordinates** of $P$, then gives us enough information to locate the point $P$ in the plane.

**Figure 2**

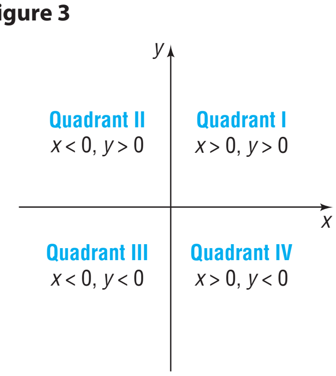



For example, to locate the point whose coordinates are $(-3, 1)$, go 3 units along the $x$-axis to the left of $O$ and then go straight up 1 unit. We **plot** this point by placing a dot at this location. See Figure 2, in which the points with coordinates $(-3, 1)$, $(-2, -3)$, $(3, -2)$, and $(3, 2)$ are plotted.

The origin has coordinates $(0, 0)$. Any point on the $x$-axis has coordinates of the form $(x, 0)$, and any point on the $y$-axis has coordinates of the form $(0, y)$.

If $(x, y)$ are the coordinates of a point $P$, then $x$ is called the ***x*-coordinate,** or **abscissa,** of $P$ and $y$ is the ***y*-coordinate,** or **ordinate,** of $P$. We identify the point $P$ by its coordinates $(x, y)$ by writing $P = (x, y)$. Usually, we will simply say, "the point $(x, y)$" rather than "the point whose coordinates are $(x, y)$."

**Figure 3**

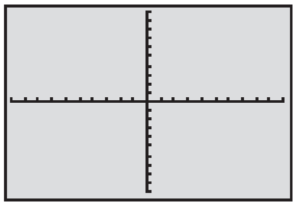



The coordinate axes divide the $xy$-plane into four sections called **quadrants,** as shown in Figure 3. In quadrant I, both the $x$-coordinate and the $y$-coordinate of all points are positive; in quadrant II, $x$ is negative and $y$ is positive; in quadrant III, both $x$ and $y$ are negative; and in quadrant IV, $x$ is positive and $y$ is negative. Points on the coordinate axes belong to no quadrant.

**Now Work** PROBLEM 11

*Named after René Descartes (1596–1650), a French mathematician, philosopher, and theologian.

Figure 4



**COMMENT** On a graphing calculator, you can set the scale on each axis. Once this has been done, you obtain the **viewing rectangle.** See Figure 4 for a typical viewing rectangle. You should now read Section B.1, *The Viewing Rectangle*, in Appendix B. ■

## 1 Use the Distance Formula

If the same units of measurement (such as inches, centimeters, and so on) are used for both the $x$-axis and $y$-axis, then all distances in the $xy$-plane can be measured using this unit of measurement.

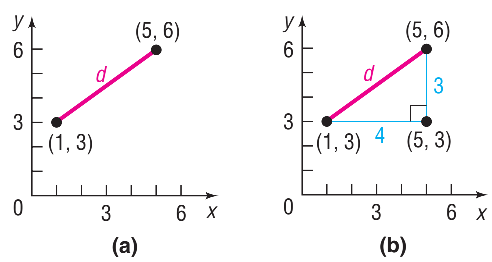
### EXAMPLE 1 Finding the Distance between Two Points

Find the distance $d$ between the points $(1, 3)$ and $(5, 6)$.

**Solution** First plot the points $(1, 3)$ and $(5, 6)$ and connect them with a straight line. See Figure 5(a). To find the length $d$, begin by drawing a horizontal line from $(1, 3)$ to $(5, 3)$ and a vertical line from $(5, 3)$ to $(5, 6)$, forming a right triangle, as shown in Figure 5(b). One leg of the triangle is of length 4 (since $|5 - 1| = 4$), and the other is of length 3 (since $|6 - 3| = 3$). By the Pythagorean Theorem, the square of the distance $d$ that we seek is

$$d^2 = 4^2 + 3^2 = 16 + 9 = 25$$
$$d = \sqrt{25} = 5$$

Figure 5

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The **distance formula** provides a straightforward method for computing the distance between two points.

**THEOREM**

**Distance Formula**

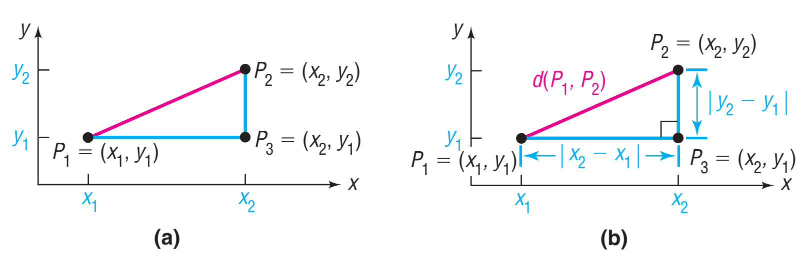
The distance between two points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$, denoted by $d(P_1, P_2)$, is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \textbf{(1)}$$

**In Words**
*To compute the distance between two points, find the difference of the x-coordinates, square it, and add this to the square of the difference of the y-coordinates. The square root of this sum is the distance.*

**Proof of the Distance Formula** Let $(x_1, y_1)$ denote the coordinates of point $P_1$ and let $(x_2, y_2)$ denote the coordinates of point $P_2$. Assume that the line joining $P_1$ and $P_2$ is neither horizontal nor vertical. Refer to Figure 6(a). The coordinates of $P_3$ are $(x_2, y_1)$. The horizontal distance from $P_1$ to $P_3$ is the absolute value of the difference of the $x$-coordinates, $|x_2 - x_1|$. The vertical distance from $P_3$ to $P_2$ is the

Figure 6



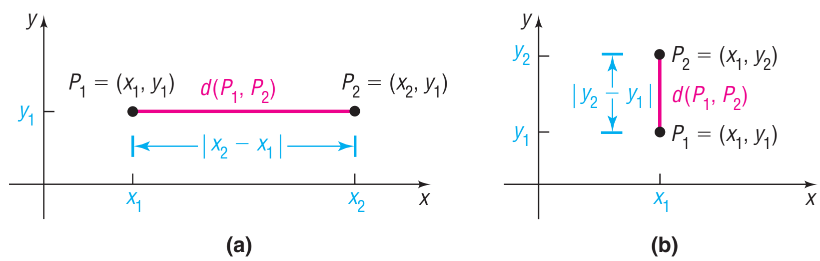
absolute value of the difference of the $y$-coordinates, $|y_2 - y_1|$. See Figure 6(b). The distance $d(P_1, P_2)$ that we seek is the length of the hypotenuse of the right triangle, so, by the Pythagorean Theorem, it follows that

$$\begin{aligned}[d(P_1, P_2)]^2 &= |x_2 - x_1|^2 + |y_2 - y_1|^2 \\ &= (x_2 - x_1)^2 + (y_2 - y_1)^2 \\ d(P_1, P_2) &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\end{aligned}$$

Now, if the line joining $P_1$ and $P_2$ is horizontal, then the $y$-coordinate of $P_1$ equals the $y$-coordinate of $P_2$; that is, $y_1 = y_2$. Refer to Figure 7(a). In this case, the distance formula (1) still works, because, for $y_1 = y_2$, it reduces to

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + 0^2} = \sqrt{(x_2 - x_1)^2} = |x_2 - x_1|$$

Figure 7




A similar argument holds if the line joining $P_1$ and $P_2$ is vertical. See Figure 7(b). ■

## EXAMPLE 2 Using the Distance Formula

Find the distance $d$ between the points $(-3, 5)$ and $(3, 2)$.

**Solution** Use the distance formula, equation (1), with $P_1 = (x_1, y_1) = (-3, 5)$ and $P_2 = (x_2, y_2) = (3, 2)$. Then

$$\begin{aligned}d = \sqrt{[3 - (-3)]^2 + (2 - 5)^2} &= \sqrt{6^2 + (-3)^2} \\ &= \sqrt{36 + 9} \\ &= \sqrt{45} \\ &= 3\sqrt{5} \approx 6.71\end{aligned}$$

●

**Now Work** PROBLEMS 15 AND 19

The distance between two points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ is never a negative number. Furthermore, the distance between two points is 0 only when the points are identical—that is, when $x_1 = x_2$ and $y_1 = y_2$. Also, because $(x_2 - x_1)^2 = (x_1 - x_2)^2$ and $(y_2 - y_1)^2 = (y_1 - y_2)^2$, it makes no difference whether the distance is computed from $P_1$ to $P_2$ or from $P_2$ to $P_1$; that is, $d(P_1, P_2) = d(P_2, P_1)$.

The introduction to this chapter mentioned that rectangular coordinates enable us to translate geometry problems into algebra problems, and vice versa. The next example shows how algebra (the distance formula) can be used to solve geometry problems.

## EXAMPLE 3 Using Algebra to Solve Geometry Problems

Consider the three points $A = (-2, 1)$, $B = (2, 3)$, and $C = (3, 1)$.

(a) Plot each point and form the triangle $ABC$.
(b) Find the length of each side of the triangle.
(c) Verify that the triangle is a right triangle.
(d) Find the area of the triangle.

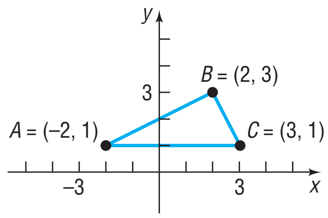
**Solution**

(a) Figure 8 shows the points $A, B, C$ and the triangle $ABC$.

(b) To find the length of each side of the triangle, use the distance formula, equation (1).

$$d(A, B) = \sqrt{[2 - (-2)]^2 + (3 - 1)^2} = \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}$$
$$d(B, C) = \sqrt{(3 - 2)^2 + (1 - 3)^2} = \sqrt{1 + 4} = \sqrt{5}$$
$$d(A, C) = \sqrt{[3 - (-2)]^2 + (1 - 1)^2} = \sqrt{25 + 0} = 5$$

**Figure 8**




(c) If the triangle is a right triangle, then the sum of the squares of the lengths of two of the sides will equal the square of the length of the third side. (Why is this sufficient?) Looking at Figure 8, it seems reasonable to conjecture that the right angle is at vertex $B$. We shall check to see whether

$$[d(A, B)]^2 + [d(B, C)]^2 = [d(A, C)]^2$$

Using the results from part (b) yields

$$[d(A, B)]^2 + [d(B, C)]^2 = (2\sqrt{5})^2 + (\sqrt{5})^2$$
$$= 20 + 5 = 25 = [d(A, C)]^2$$

It follows from the converse of the Pythagorean Theorem that triangle $ABC$ is a right triangle.

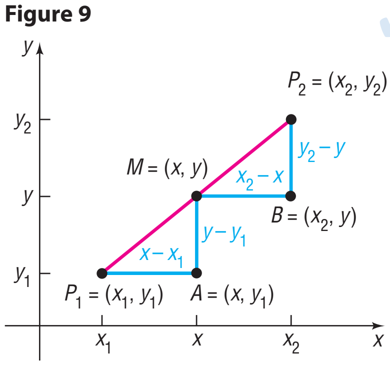
(d) Because the right angle is at vertex $B$, the sides $AB$ and $BC$ form the base and height of the triangle. Its area is

$$\text{Area} = \frac{1}{2}(\text{Base})(\text{Height}) = \frac{1}{2}(2\sqrt{5})(\sqrt{5}) = 5 \text{ square units}$$

**Now Work** PROBLEM 29

## 2 Use the Midpoint Formula

**Figure 9**




We now derive a formula for the coordinates of the **midpoint of a line segment.** Let $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ be the endpoints of a line segment, and let $M = (x, y)$ be the point on the line segment that is the same distance from $P_1$ as it is from $P_2$. See Figure 9. The triangles $P_1AM$ and $MBP_2$ are congruent. [Do you see why? Angle $AP_1M$ = angle $BMP_2$,* angle $P_1MA$ = angle $MP_2B$, and $d(P_1, M) = d(M, P_2)$ is given. Thus we have angle–side–angle.] Hence, corresponding sides are equal in length. That is,

$$x - x_1 = x_2 - x \quad \text{and} \quad y - y_1 = y_2 - y$$
$$2x = x_1 + x_2 \qquad 2y = y_1 + y_2$$
$$x = \frac{x_1 + x_2}{2} \qquad y = \frac{y_1 + y_2}{2}$$

**THEOREM**

**Midpoint Formula**

The midpoint $M = (x, y)$ of the line segment from $P_1 = (x_1, y_1)$ to $P_2 = (x_2, y_2)$ is

$$M = (x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \qquad \textbf{(2)}$$

**In Words**

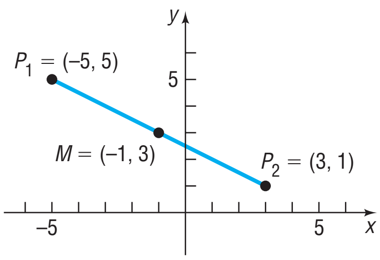
*To find the midpoint of a line segment, average the x-coordinates of the endpoints, and average the y-coordinates of the endpoints.*

*A postulate from geometry states that the transversal $\overline{P_1P_2}$ forms congruent corresponding angles with the parallel line segments $\overline{P_1A}$ and $\overline{MB}$.

**EXAMPLE 4** **Finding the Midpoint of a Line Segment**

Find the midpoint of the line segment from $P_1 = (-5, 5)$ to $P_2 = (3, 1)$. Plot the points $P_1$ and $P_2$ and the midpoint.

**Figure 10**




**Solution** Apply the midpoint formula (2) using $x_1 = -5$, $y_1 = 5$, $x_2 = 3$, and $y_2 = 1$. Then the coordinates $(x, y)$ of the midpoint $M$ are

$$x = \frac{x_1 + x_2}{2} = \frac{-5 + 3}{2} = -1 \quad \text{and} \quad y = \frac{y_1 + y_2}{2} = \frac{5 + 1}{2} = 3$$

That is, $M = (-1, 3)$. See Figure 10.

**Now Work** PROBLEM 35

## F.1 Assess Your Understanding

**'Are You Prepared?'** *Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.*

1. On the real number line the origin is assigned the number ________. (p. A4)
2. If $-3$ and 5 are the coordinates of two points on the real number line, the distance between these points is ________. (p. A6)
3. If 3 and 4 are the legs of a right triangle, the hypotenuse is ________. (pp. A13–A14)
4. Use the converse of the Pythagorean Theorem to show that a triangle whose sides are of lengths 11, 60, and 61 is a right triangle. (p. A14)
5. State the formula for the area $A$ of a triangle whose base is $b$ and whose altitude is $h$. (p. A15)
6. State the three cases for which two triangles are congruent. (p. A16)

### Concepts and Vocabulary

7. If $(x, y)$ are the coordinates of a point $P$ in the $xy$-plane, then $x$ is called the ________ of $P$, and $y$ is the ________ of $P$.
8. The coordinate axes divide the $xy$-plane into four sections called ________.
9. The distance $d$ between two points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ is $d =$ ________.
10. If three distinct points $P$, $Q$, and $R$ all lie on a line, and if $d(P, Q) = d(Q, R)$, then $Q$ is called the ________ of the line segment from $P$ to $R$.

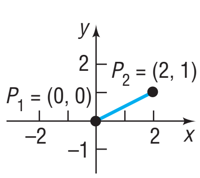
### Skill Building

*In Problems 11 and 12, plot each point in the xy-plane. Tell in which quadrant or on what coordinate axis each point lies.*

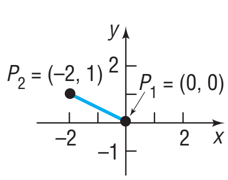
11. (a) $A = (-3, 2)$ (b) $B = (6, 0)$ (c) $C = (-2, -2)$ (d) $D = (6, 5)$ (e) $E = (0, -3)$ (f) $F = (6, -3)$
12. (a) $A = (1, 4)$ (b) $B = (-3, -4)$ (c) $C = (-3, 4)$ (d) $D = (4, 1)$ (e) $E = (0, 1)$ (f) $F = (-3, 0)$
13. Plot the points $(2, 0)$, $(2, -3)$, $(2, 4)$, $(2, 1)$, and $(2, -1)$. Describe the set of all points of the form $(2, y)$, where $y$ is a real number.
14. Plot the points $(0, 3)$, $(1, 3)$, $(-2, 3)$, $(5, 3)$, and $(-4, 3)$. Describe the set of all points of the form $(x, 3)$, where $x$ is a real number.

*In Problems 15–28, find the distance $d(P_1, P_2)$ between the points $P_1$ and $P_2$.*

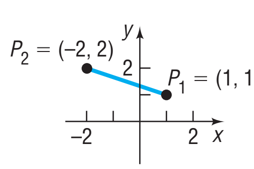
15.



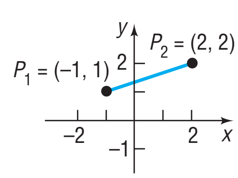

16.




17.




18.



**19.** $P_1 = (3, -4)$; $P_2 = (5, 4)$

**20.** $P_1 = (-1, 0)$; $P_2 = (2, 4)$

**21.** $P_1 = (-3, 2)$; $P_2 = (6, 0)$

**22.** $P_1 = (2, -3)$; $P_2 = (4, 2)$

**23.** $P_1 = (4, -2)$; $P_2 = (-2, -5)$

**24.** $P_1 = (-4, -3)$; $P_2 = (6, 2)$

**25.** $P_1 = (-0.2, 0.3)$; $P_2 = (2.3, 1.1)$

**26.** $P_1 = (1.2, 2.3)$; $P_2 = (-0.3, 1.1)$

**27.** $P_1 = (a, b)$; $P_2 = (0, 0)$

**28.** $P_1 = (a, a)$; $P_2 = (0, 0)$

*In Problems 29–34, plot each point and form the triangle ABC. Verify that the triangle is a right triangle. Find its area.*

**29.** $A = (-2, 5)$; $B = (1, 3)$; $C = (-1, 0)$

**30.** $A = (-2, 5)$; $B = (12, 3)$; $C = (10, -11)$

**31.** $A = (-5, 3)$; $B = (6, 0)$; $C = (5, 5)$

**32.** $A = (-6, 3)$; $B = (3, -5)$; $C = (-1, 5)$

**33.** $A = (4, -3)$; $B = (0, -3)$; $C = (4, 2)$

**34.** $A = (4, -3)$; $B = (4, 1)$; $C = (2, 1)$

*In Problems 35–44, find the midpoint of the line segment joining the points $P_1$ and $P_2$.*

**35.** $P_1 = (3, -4)$; $P_2 = (5, 4)$

**36.** $P_1 = (-2, 0)$; $P_2 = (2, 4)$

**37.** $P_1 = (-3, 2)$; $P_2 = (6, 0)$

**38.** $P_1 = (2, -3)$; $P_2 = (4, 2)$

**39.** $P_1 = (4, -2)$; $P_2 = (-2, -5)$

**40.** $P_1 = (-4, -3)$; $P_2 = (2, 2)$

**41.** $P_1 = (-0.2, 0.3)$; $P_2 = (2.3, 1.1)$

**42.** $P_1 = (1.2, 2.3)$; $P_2 = (-0.3, 1.1)$

**43.** $P_1 = (a, b)$; $P_2 = (0, 0)$

**44.** $P_1 = (a, a)$; $P_2 = (0, 0)$

## Applications and Extensions

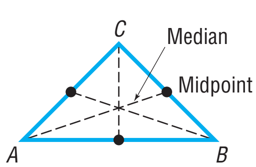
**45.** Find all points having an $x$-coordinate of 2 whose distance from the point $(-2, -1)$ is 5.

**46.** Find all points having a $y$-coordinate of $-3$ whose distance from the point $(1, 2)$ is 13.

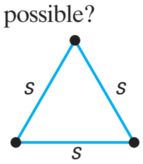
**47.** Find all points on the $x$-axis that are 5 units from the point $(4, -3)$.

**48.** Find all points on the $y$-axis that are 5 units from the point $(4, 4)$.

**49. Geometry** The **medians** of a triangle are the line segments from each vertex to the midpoint of the opposite side (see the figure). Find the lengths of the medians of the triangle with vertices at $A = (0, 0)$, $B = (6, 0)$, and $C = (4, 4)$.




**50. Geometry** An **equilateral triangle** is one in which all three sides are of equal length. If two vertices of an equilateral triangle are $(0, 4)$ and $(0, 0)$, find the third vertex. How many of these triangles are possible?




**51. Geometry** Find the midpoint of each diagonal of a square with side of length $s$. Draw the conclusion that the diagonals of a square intersect at their midpoints. [**Hint:** Use $(0, 0)$, $(0, s)$, $(s, 0)$, and $(s, s)$ as the vertices of the square.]

**52. Geometry** Verify that the points $(0, 0)$, $(a, 0)$, and $\left(\frac{a}{2}, \frac{\sqrt{3}a}{2}\right)$ are the vertices of an equilateral triangle. Then show that the midpoints of the three sides are the vertices of a second equilateral triangle (refer to Problem 50).

*In Problems 53–56, find the length of each side of the triangle determined by the three points $P_1$, $P_2$, and $P_3$. State whether the triangle is an isosceles triangle, a right triangle, neither of these, or both. (An* **isosceles triangle** *is one in which at least two of the sides are of equal length.)*

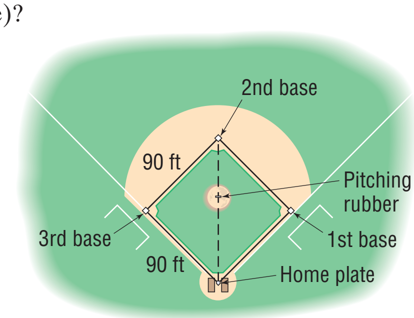
**53.** $P_1 = (2, 1)$; $P_2 = (-4, 1)$; $P_3 = (-4, -3)$

**54.** $P_1 = (-1, 4)$; $P_2 = (6, 2)$; $P_3 = (4, -5)$

**55.** $P_1 = (-2, -1)$; $P_2 = (0, 7)$; $P_3 = (3, 2)$

**56.** $P_1 = (7, 2)$; $P_2 = (-4, 0)$; $P_3 = (4, 6)$

**57. Baseball** A major league baseball "diamond" is actually a square 90 feet on a side (see the figure). What is the distance directly from home plate to second base (the diagonal of the square)?



**58. Little League Baseball** The layout of a Little League playing field is a square 60 feet on a side. How far is it directly from home plate to second base (the diagonal of the square)?

***Source:*** *Little League Baseball, Official Regulations and Playing Rules, 2012.*

**59. Baseball** Refer to Problem 57. Overlay a rectangular coordinate system on a major league baseball diamond so that the origin is at home plate, the positive $x$-axis lies in the direction from home plate to first base, and the positive $y$-axis lies in the direction from home plate to third base.

(a) What are the coordinates of first base, second base, and third base? Use feet as the unit of measurement.

(b) If the right fielder is located at $(310, 15)$, how far is it from there to second base?

(c) If the center fielder is located at $(300, 300)$, how far is it from there to third base?

**60. Little League Baseball** Refer to Problem 58. Overlay a rectangular coordinate system on a Little League baseball diamond so that the origin is at home plate, the positive $x$-axis lies in the direction from home plate to first base, and the positive $y$-axis lies in the direction from home plate to third base.

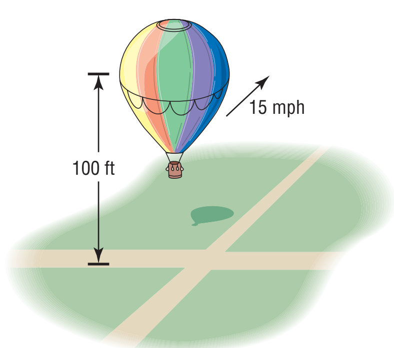
(a) What are the coordinates of first base, second base, and third base? Use feet as the unit of measurement.

(b) If the right fielder is located at $(180, 20)$, how far is it from there to second base?

(c) If the center fielder is located at $(220, 220)$, how far is it from there to third base?

**61. Distance between Moving Objects** A Ford Focus and a Mack truck leave an intersection at the same time. The Focus heads east at an average speed of 30 miles per hour, while the truck heads south at an average speed of 40 miles per hour. Find an expression for their distance apart $d$ (in miles) at the end of $t$ hours.

**62. Distance of a Moving Object from a Fixed Point** A hot-air balloon, headed due east at an average speed of 15 miles per hour and at a constant altitude of 100 feet, passes over an intersection (see the figure). Find an expression for the distance $d$ (measured in feet) from the balloon to the intersection $t$ seconds later.

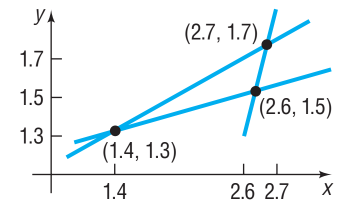



**63. Drafting Error** When a draftsman draws three lines that are to intersect at one point, the lines may not intersect as intended and subsequently will form an **error triangle.** If this error triangle is long and thin, one estimate for the location of the desired point is the midpoint of the shortest side. The figure shows one such error triangle.

(a) Find an estimate for the desired intersection point.

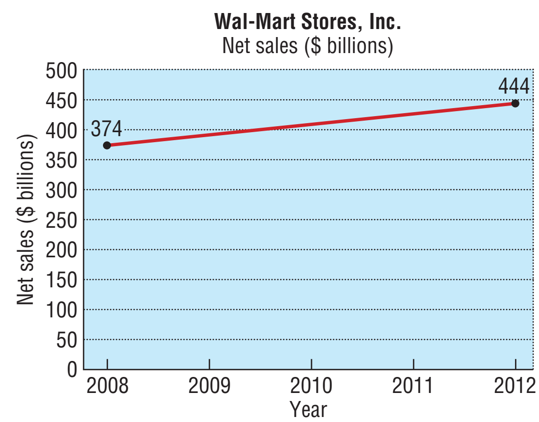
(b) Find the length of the median for the midpoint found in part (a). See Problem 49.

***Source:*** *www.uwgb.edu/DutchS/structge/s100.htm*




**64. Net Sales** The figure illustrates how net sales of Wal-Mart Stores, Inc., grew from 2008 through 2012. Use the midpoint formula to estimate the net sales of Wal-Mart Stores, Inc., in 2010. How does your result compare to the reported value of \$405 billion?

***Source:*** *Wal-Mart Stores, Inc., 2012 Annual Report*




**65. Poverty Threshold** Poverty thresholds are determined by the U.S. Census Bureau. A poverty threshold represents the minimum annual household income for a family not to be considered poor. In 2004, the poverty threshold for a family of four with two children under the age of 18 years was \$19,157. In 2012, the poverty threshold for a family of four with two children under the age of 18 years was \$23,283. Assuming poverty thresholds increase in a straight-line fashion, use the midpoint formula to estimate the poverty threshold of a family of four with two children under the age of 18 in 2008. How does your result compare to the actual poverty threshold in 2008 of \$21,834?

***Source:*** *U.S. Census Bureau*

**66. Horizontal and Vertical Shifts** Suppose that $A = (2, 5)$ are the coordinates of a point in the $xy$-plane.

(a) Find the coordinates of the point if $A$ is shifted 3 units to the right and 2 units down.

(b) Find the coordinates of the point if $A$ is shifted 2 units to the left and 8 units up.

**67. Completing a Line Segment** Plot the points $A = (-1, 8)$ and $M = (2, 3)$ in the $xy$-plane. If $M$ is the midpoint of a line segment $AB$, find the coordinates of $B$.

## Discussion and Writing

68. Write a paragraph that describes a Cartesian plane. Then write a second paragraph that describes how to plot points in the Cartesian plane. Your paragraphs should include the terms *coordinate axes*, *ordered pair*, *coordinates*, *plot*, *x-coordinate*, and *y-coordinate*.

## 'Are You Prepared?' Answers

**1.** 0 **2.** 8 **3.** 5 **4.** $11^2 + 60^2 = 61^2$ **5.** $A = \frac{1}{2}bh$ **6.** Angle–side–angle; side–side–side; side–angle–side

# F.2 Graphs of Equations in Two Variables; Intercepts; Symmetry

**PREPARING FOR THIS SECTION** *Before getting started, review the following:*

- Solving Equations (Appendix A, Section A.8, pp. A63–A69)

**Now Work** the **'Are You Prepared?'** problems on page 16.

**OBJECTIVES**



## 1 Graph Equations by Plotting Points

An **equation in two variables,** say $x$ and $y$, is a statement in which two expressions involving $x$ and $y$ are equal. The expressions are called the **sides** of the equation. Since an equation is a statement, it may be true or false, depending on the value of the variables. Any values of $x$ and $y$ that result in a true statement are said to **satisfy** the equation.

For example, the following are all equations in two variables $x$ and $y$:

$$x^2 + y^2 = 5 \qquad 2x - y = 6 \qquad y = 2x + 5 \qquad x^2 = y$$

The first of these, $x^2 + y^2 = 5$, is satisfied for $x = 1, y = 2$, since $1^2 + 2^2 = 1 + 4 = 5$. Other choices of $x$ and $y$, such as $x = -1$, $y = -2$, also satisfy this equation. It is not satisfied for $x = 2$ and $y = 3$, since $2^2 + 3^2 = 4 + 9 = 13 \neq 5$.

The **graph of an equation in two variables** $x$ and $y$ consists of the set of points in the $xy$-plane whose coordinates $(x, y)$ satisfy the equation.

### EXAMPLE 1 Determining Whether a Point Is on the Graph of an Equation

Determine if the following points are on the graph of the equation $2x - y = 6$.

(a) $(2, 3)$ (b) $(2, -2)$

**Solution**

(a) For the point $(2, 3)$, check to see whether $x = 2$, $y = 3$ satisfies the equation $2x - y = 6$.

$$2x - y = 2(2) - 3 = 4 - 3 = 1 \neq 6$$

The equation is not satisfied, so the point $(2, 3)$ is not on the graph.